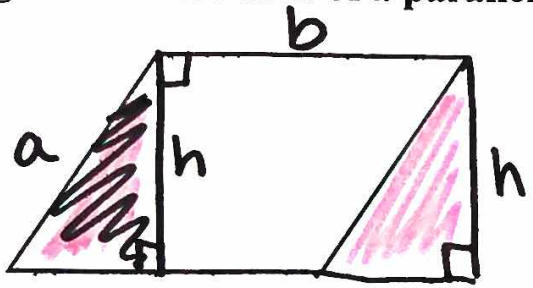
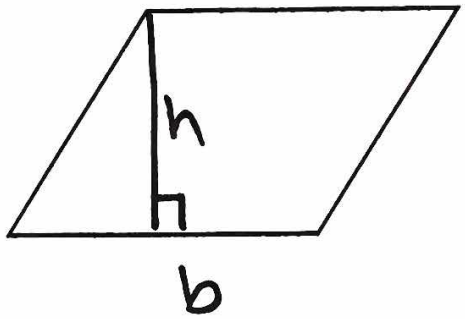
Deriving formula for area of a parallelogram:



Area of a Parallelogram

The area of a parallelogram with base b and height h is:

$$b \cdot h$$

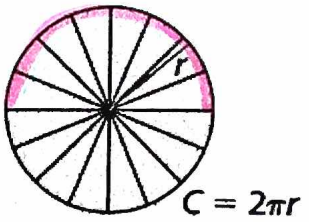


Name: Date: Period:

Area of Circles, Sectors, and Segments

Lesson Objective IWBAT find area of ⊙s, sectors, and segments of ⊙s.

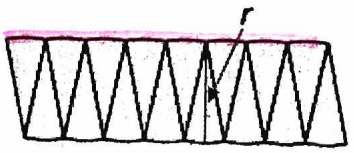
You can divide a circle into congruent sections and rearrange the sections to form a figure that approximates a parallelogram. Increasing the number of congruent sections increases the figure's resemblance to a parallelogram.



The base of the parallelogram that the figure approaches is half of the circumference, so the base of the parallelogram is $\frac{1}{2}(2\pi r) = \pi r$

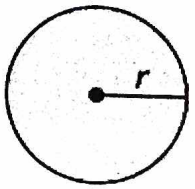
The height of the parallelogram is the radius, r

So, the area of the parallelogram is $(\pi r)(r) = \pi r^2$



Area of a Circle

The area of a circle is πr^2, where r is the radius of the circle.



Examples:

1. Find the area of a circle with a radius of 2.5 cm.

$\pi(2.5)^2 = 6.25\pi \text{ cm}^2$

$\approx 19.63 \text{ cm}^2$

2. Find the diameter of a circle with an area of 113.1 square cm.

$113.1 = \pi r^2$

$r^2 = 36.0008$

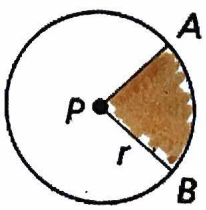
$r \approx 6.00 \longrightarrow$ $d \approx 12.00$ cm

Sector of a Circle: A sector of a circle is the region bounded by two radii of the circle and their intercepted arc.

In the diagram below, sector APB is bounded by $\overline{AP}$, $\overline{BP}$ and $\overset{\frown}{AB}$.

Area of a Sector

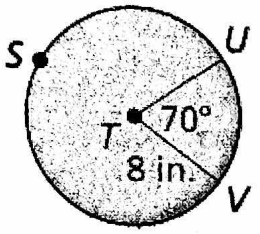
The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc and 360°.



$$\frac{\text{arc measure}}{360} = \frac{\text{sector area}}{\pi r^2}$$

mple:

3. Find the areas of the small sector formed by $\angle UTV$.

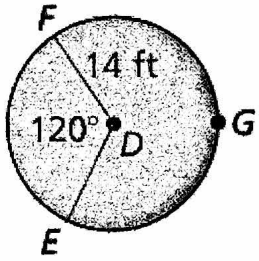


$$\frac{70^\circ}{360^\circ} = \frac{S}{\pi(8)^2}$$

$$360S = 448\pi$$

$$S = \frac{112\pi}{9}\ in^2 \approx 39.10\ in^2$$

4. Find the area of the large sector formed by $\angle FDE$.

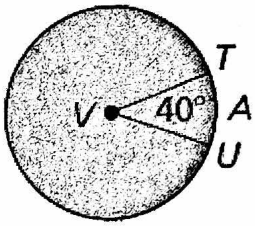


$$\frac{240}{360} = \frac{S}{\pi(14)^2}$$

$$3S = 392\pi$$

$$S = \frac{392\pi}{3}\ ft^2 \approx 410.50\ ft^2$$

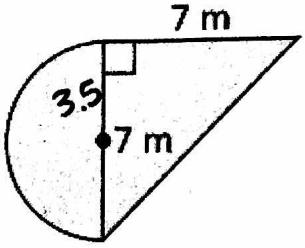
5. Find the area of $\odot V$.



$$\frac{40}{360} = \frac{35}{\text{area } \odot}$$

$$\text{area } \odot = 315\ m^2$$

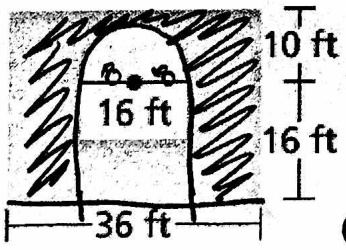
6. Find the area of the figure.



$$\frac{\pi(3.5)^2}{2} + \frac{(7)(7)}{2}$$

$$\frac{49\pi}{8} + \frac{49}{2} \approx 43.74\ m^2$$

7. A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?



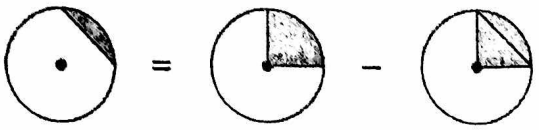
$\text{big rect} = (36)(26) = 936$

$\text{semi } \odot = \frac{\pi 8^2}{2} = 32\pi \approx 100.53$

$\text{square} = 16^2 = 256$

$936 - 100.53 - 256 \approx 579.47\ ft^2$

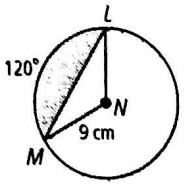
Segment of a circle:



area of segment = area of sector − area of triangle

$\frac{1}{2}ab\sin C$

8. Find the area of segment LNM in simplest form and to the nearest hundredth.



sector: $\frac{120}{360} = \frac{s}{\pi 9^2} \rightarrow 3s = 81\pi \rightarrow s = 27\pi \approx 84.82$

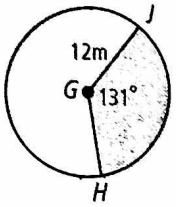
Δ: $\frac{1}{2}(9)(9)\sin 120 \approx 35.07$

segment: $84.82 - 35.07$ $\boxed{\approx 49.75\ cm^2}$

Extra Examples:

9. Find the area of each sector. Give answers in terms of π and rounded to the nearest hundredth.

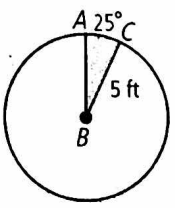
a. sector HGJ



$\frac{131}{360} = \frac{s}{\pi 12^2}$

$s = \frac{262\pi}{5}$ $\boxed{\approx 164.62\ m^2}$

b. sector ABC



$\frac{25}{360} = \frac{s}{\pi 5^2}$

$s = \frac{125\pi}{72}$ $\boxed{\approx 5.45\ ft^2}$

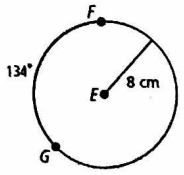
10. A windshield wiper blade is 18 in long. To the nearest square inch, what is the area covered by the blade as it rotates through an angle of 122°?

$\frac{122}{360} = \frac{s}{\pi 18^2}$ $s = \frac{549\pi}{5} \approx 344.95\ in^2$

$\boxed{\approx 345\ in^2}$

11. Find each arc length. Give answers in terms of π and rounded to the nearest hundredth.

a. Length of arc FG



$\frac{134}{360} = \frac{a}{2\pi \cdot 8}$

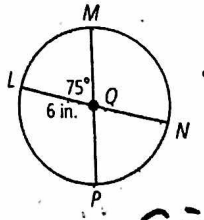
$a = \frac{268\pi}{45}$ $\boxed{\approx 18.71\ cm}$

b. an arc with measure 62° in a circle with radius 2 m.

$\frac{62}{360} = \frac{a}{2\pi(2)}$

$a = \frac{31\pi}{45}$ $\boxed{\approx 2.16\ m}$

12. Area of sector LQM and length of arc NP



$\frac{75}{360} = \frac{s}{\pi 6^2}$

$s = \frac{15\pi}{2}$ $\boxed{\approx 23.56\ in^2}$

$\frac{75}{360} = \frac{a}{2\pi \cdot 6}$

$a = \frac{5\pi}{2}$ $\boxed{\approx 7.85\ in}$